Available online at www.sciencedirect.com
ScienceDirect
Journal of Sound and Vibration 306 (2007) 790–802
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

The Poisson's loss factor of solid viscoelastic materials

T. Pritz*

Acoustics Laboratory, Szikkti Labs, 1301 Budapest, Pf. 81, Hungary

Received 3 November 2006; received in revised form 21 June 2007; accepted 26 June 2007

Abstract

The complex Poisson's ratio plays an important role in characterizing the linear dynamic behaviour of solid materials, and occurs in a number of equations used for acoustical and vibration calculus. The ratio of the imaginary part to the real part of complex Poisson's ratio is referred to as Poisson's loss factor. The magnitude of the Poisson's loss factor is investigated in this paper for homogeneous, isotropic, linear solid viscoelastic materials with positive Poisson's ratio. The relation of the Poisson's loss factor to the material damping is determined. It is shown that the magnitude of the Poisson's loss factor is approximately proportional to the difference between the shear and bulk loss factors, and is a rational fractional function of the dynamic Poisson's ratio. In addition, relationships are developed which enable one to determine the approximate magnitude of the Poisson's loss factor from knowledge only of the shear loss factor and the dynamic Poisson's ratio. It is shown that the Poisson's loss factor is smaller than the shear loss factor usually by one order of magnitude at least. Moreover, it is pointed out that the Poisson's loss factor of a high loss and a low loss material may be about the same. Experimental data on two rubbers and a hard plastic are presented to verify the theoretical conclusions.

1. Introduction

The Poisson's ratio is defined as the ratio of the lateral strain to the axial strain in a linear solid body, e.g., a cylinder, loaded uniaxially, and this ratio is a real number in case of perfect elasticity for either static or dynamic loading. In contrast, with dynamic loading of a real, i.e., viscoelastic solid, the strain-to-strain ratio can be considered as a complex number due to the fact that the lateral strain lags behind the axial strain as a result of damping in the material. This complex number is referred to as complex Poisson's ratio [1,2]. The real part is known as dynamic Poisson's ratio, the imaginary part is related to the strain lag, and the ratio of the imaginary part to the real part is named Poisson's loss factor [3]. The complex Poisson's ratio describes the strain-to-strain ratio in the frequency domain, both the real and imaginary parts depend on the frequency [1–4].

The complex Poisson's ratio is the counterpart of the complex modulus of elasticity widely used to characterize the dynamic elastic and damping properties of solid materials. While vast theoretical and experimental knowledge have been accumulated on the complex moduli (shear, Young's, etc.) [1,5,6], the information on the complex Poisson's ratio, especially its loss part, is less complete and ambiguous. There is solid theoretical knowledge on the magnitude of dynamic Poisson's ratio, which may be only between −1 and

*Tel.: +36 1 326 0051; fax: +36 1 3687 626.
E-mail address: tampri@eik.bme.hu

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2007.06.016

0.5 with homogeneous, isotropic materials [7]. In addition, the dynamic Poisson's ratio is more or less known from experiments for materials of different kind. In contrast, the magnitude of the Poisson's loss factor is not known from the theoretical studies [8,9], and its relation to the material damping is not clear. While a modulus loss factor, e.g., the shear one is directly related to the damping, the experiments have revealed that the Poisson's loss factor can be extremely low even if a material has high damping capacity [8]. Experimental data on the Poisson's loss factor are scanty, and most of them concern solid polymeric materials, especially the hard plastics [1,8–11] and some sound absorbing foams [12]. The available experimental data show that the Poisson's loss factor is always low, the measured magnitudes are of the order of 10^{-3}–10^{-2} [1,8–12], or it has been found to be zero for some materials [13–16] in spite of careful experiments. Referring to its smallness, the Poisson's loss factor is often neglected, and the dynamic Poisson's ratio is considered as a single, frequency independent number, however, both assumptions violate solid theoretical establishments [2–4,8,9].

The magnitude of the Poisson's loss factor is investigated in this paper for homogeneous, isotropic, linear solid viscoelastic materials. For the sake of simplicity, only the materials with positive Poisson's ratio are considered. The essential aim is to determine the magnitude of the Poisson's loss factor and to clear up its relation to the material damping. The solid polymers, especially the rubbers and hard plastics, referred usually as viscoelastic materials, are in the focus, but the theory and conclusions concern all real solids within the linear range of dynamic behaviour. The motivation of this research, beyond the scientific interest, is that the knowledge of the Poisson's loss factor is required for both the complete characterization of dynamic behaviour of materials [1,2,17,18], and the correct acoustical and vibration calculus [19–21].

2. Theory

2.1. The complex Poisson's ratio-definitions

For defining the complex Poisson's ratio and the respective loss factor, consider the strain state of a cylindrical solid body subjected to direct uniaxial dynamic stress. The axial strain, ε_x, of the cylinder is assumed to obey a harmonic function, which is given here in complex form as:

$$\varepsilon_x(t) = \hat{\varepsilon}_x e^{j\omega t}, \tag{1}$$

where t is the time, $\hat{\varepsilon}_x$ is the amplitude of the axial strain, $j = \sqrt{-1}$ is the imaginary unit, $\omega = 2\pi f$, f is the frequency in Hz. In addition, it is assumed that the strain is small enough, i.e., the dynamic behaviour of the material is linear, and then the lateral strain also obeys a harmonic function of ω frequency. Nevertheless, it is reasonable that in case of a real solid the lateral strain cannot vary simultaneously with, but lags behind the axial strain by a Δt time due to the damping in the material. Consequently, the lateral strain can be written as

$$\varepsilon_y(t) = \hat{\varepsilon}_y e^{j\omega(t-\Delta t)} = \hat{\varepsilon}_y e^{j(\omega t - \delta_v)}, \tag{2}$$

where $\hat{\varepsilon}_y$ is the amplitude of the lateral strain, and δ_v is the phase lag:

$$\delta_v = \omega \Delta t. \tag{3}$$

The ratio of the lateral strain to the axial strain results in a complex number referred to as the *complex Poisson's ratio*

$$\bar{v}(j\omega) = \frac{\varepsilon_y}{\varepsilon_x} = \frac{\hat{\varepsilon}_y}{\hat{\varepsilon}_x} e^{-j\delta_v} = \frac{\hat{\varepsilon}_y}{\hat{\varepsilon}_x}(\cos\delta_v - j\sin\delta_v) = v_d(\omega) - jv_l(\omega) = v_d(\omega)\left[1 - j\eta_v(\omega)\right], \tag{4}$$

where the overbar denotes the complex value, v_d is the *dynamic Poisson's ratio*, v_l is the *loss component,* and η_v is the *Poisson's loss factor* defined as

$$\eta_v(\omega) = \frac{v_l(\omega)}{v_d(\omega)} = \mathrm{tg}\delta_v(\omega). \tag{5}$$

In addition, the *absolute Poisson's ratio* is useful to define as:

$$\left|\bar{v}(j\omega)\right| = \frac{\hat{\varepsilon}_y}{\hat{\varepsilon}_x} = (v_d^2 + v_l^2)^{1/2} = v_d(1 + \eta_v^2)^{1/2}. \tag{6}$$

It is clear that the Poisson's loss factor is zero in case of perfect elasticity, and then Eqs. (4) and (6) yield the *Poisson's ratio*, v, which is a real, frequency independent number:

$$\bar{v}(\mathrm{j}\omega) = \left|\bar{v}(\mathrm{j}\omega)\right| = v_d = v. \tag{7}$$

The complex Poisson's ratio describes the strain-to-strain ratio in the frequency domain, and both the real and imaginary parts inevitably depend on the frequency [2–4]. The dynamic Poisson's ratio of solid viscoelastic materials is known to decrease monotonically with increasing frequency while the respective loss factor passes through a maximum [1–4]. Moreover, it is known that the decrease in $v_d(\omega)$ is related to the magnitude of the Poisson's loss factor, namely the higher η_v, the larger the decrease in $v_d(\omega)$, as it is formulated by the local Kramers–Kronig relations [4]:

$$\eta_v(\omega) \approx -\frac{\pi}{2}\frac{\mathrm{d}[\log\, v_d(\omega)]}{\mathrm{d}[\log\, \omega]}. \tag{8}$$

This equation is approximate one, but its accuracy is better than 10% if the slope of frequency variations of the components of the complex Poisson's ratio plotted in a log–log system is smaller than 0.35 [22]. This condition is usually satisfied, some examples for the frequency variations of $v_d(\omega)$ and $\eta_v(\omega)$ will be shown in the experimental part of this paper.

2.2. Magnitude of the Poisson's loss factor

It is known from the theory of elasticity that the Poisson's ratio of a homogeneous, isotropic, linear solid material cannot be arbitrary, but has bounds, namely: $-1 < v \leqslant 0.5$ [7]. The negative Poisson's ratio is scanty [23], the majority of the common solid materials have positive Poisson's ratio, and only these materials are considered in this work. The bounds on v are evidently valid for the dynamic Poisson's ratio too, i.e., $0 \leqslant v_d \leqslant 0.5$, because v_d governs the dynamic strain-to-strain ratio if the material specimen is perfectly elastic (Eq. (7)). In contrast to v_d, the magnitude of the Poisson's loss factor is not known from theoretical works, which concerned only the comparison of the loss factors. It has been found that η_v is the smallest among the modulus loss factors [8], but the relation of the Poisson's loss factor to the material damping has not been investigated.

In order to determine the magnitude the Poisson's loss factor, and to clear up the relation to the material damping, the relation between the complex Poisson's ratio and the complex moduli is investigated. A homogeneous, isotropic solid material is known to have two independent complex moduli, namely the complex shear modulus, $\bar{G}$, and the bulk modulus, $\bar{B}$, and, therefore, their relation to the complex Poisson's ratio is considered here. The respective relationship is [2]

$$\bar{v}(\mathrm{j}\omega) = \frac{3\bar{B}(\mathrm{j}\omega) - 2\bar{G}(\mathrm{j}\omega)}{6\bar{B}(\mathrm{j}\omega) + 2\bar{G}(\mathrm{j}\omega)}, \tag{9}$$

where

$$\bar{G}(\mathrm{j}\omega) = G_d(\omega) + \mathrm{j}G_l(\omega) = G_d(\omega)[1 + \mathrm{j}\eta_G(\omega)] \tag{10}$$

and

$$\bar{B}(\mathrm{j}\omega) = B_d(\omega) + \mathrm{j}B_l(\omega) = B_d(\omega)[1 + \mathrm{j}\eta_B(\omega)], \tag{11}$$

in which G_d and B_d are the dynamic shear modulus and bulk modulus, respectively, G_l and B_l are the loss moduli, moreover η_G and η_B are the modulus loss factors defined as:

$$\eta_G(\omega) = \frac{G_l(\omega)}{G_d(\omega)}, \tag{12}$$

$$\eta_B(\omega) = \frac{B_l(\omega)}{B_d(\omega)}. \tag{13}$$

The magnitude of the Poisson's loss factor could be derived from Eq. (9), but it is more suitable to determine the bulk loss factor as a first step of the derivation. The rearrangement of Eq. (9) yields the complex bulk modulus:

$$\bar{B}(\mathrm{j}\omega) = \frac{2}{3}\bar{G}(\mathrm{j}\omega)\frac{1+\bar{v}(\mathrm{j}\omega)}{1-2\bar{v}(\mathrm{j}\omega)} \tag{14}$$

and the separation of Eq. (14) into real and imaginary parts results in:

$$\eta_B = \frac{[(1+v_d)(1-2v_d) - 2(v_d\eta_v)^2]\eta_G - 3v_d\eta_v}{(1+v_d)(1-2v_d) - 2(v_d\eta_v)^2 + 3v_d\eta_v\eta_G}. \tag{15}$$

The rearrangement of Eq. (15) and some mathematical manipulations give an equation of second degree for η_v as:

$$\frac{2}{3}\frac{\eta_G - \eta_B}{1+\eta_G\eta_B}v_d\eta_v^2 + \eta_v - \frac{\eta_G - \eta_B}{1+\eta_G\eta_B}\frac{(1+v_d)(1-2v_d)}{3v_d} = 0. \tag{16}$$

The physically meaningful root of Eq. (16) is:

$$\eta_v = \frac{-1 + \left\{1 + \frac{8}{9}\left(\frac{\eta_G - \eta_B}{1+\eta_G\eta_B}\right)^2 (1+v_d)(1-2v_d)\right\}^{1/2}}{\frac{4}{3}\frac{\eta_G - \eta_B}{1+\eta_G\eta_B}v_d}. \tag{17}$$

It can be seen that the magnitude of the Poisson's loss factor is related to both the modulus loss factors, i.e., the material damping, and the dynamic Poisson's ratio.

Eq. (17) can be written in a simpler form by taking the first-order approximation of the square root. This approximation can be done if:

$$\eta_G^2\frac{8}{9}\left(\frac{1-\eta_B/\eta_G}{1+\eta_G\eta_B}\right)^2(1+v_d)(1-2v_d) \ll 1, \tag{18}$$

where $0 \leqslant 1 - \eta_B/\eta_G \leqslant 1$, because the bulk loss factor is smaller than the shear one [8,9], moreover $0 \leqslant (1+v_d)(1-2v_d) \leqslant 1$ for $0 \leqslant v_d \leqslant 0.5$. It follows that Eq. (18) holds true if:

$$\eta_G^2 \ll 1, \quad \text{i.e.,} \quad \eta_G < 0.3 \tag{19}$$

and then

$$\eta_G\eta_B \ll 1 \tag{20}$$

is valid too. Using these approximations, Eq. (17) yields:

$$\eta_v \approx (\eta_G - \eta_B)\frac{(1+v_d)(1-2v_d)}{3v_d}. \tag{21}$$

Before going further, the validity of Eq. (21) with respect to real solid materials is useful to study. The basic assumption required to derive Eq. (21) that the shear loss is as low as $\eta_G < 0.3$, holds true for the majority if not all stiff structural materials (metals, ceramics, composites, etc.) and even the hard plastics at room temperature. Nevertheless, it is easy to show that Eq. (21) may be valid for the high loss rubbery materials too. Namely, it is known that the rubbers are usually *nearly incompressible*, i.e., the dynamic Poisson's ratio is close to 0.5, and therefore, Eq. (18) holds true due to the member $(1-2v_d)$. Moreover, it is clear that η_B is inevitably very low in the case of near-incompressibility, because $B_d \to \infty$ as $v_d \to 0.5$, therefore, $\eta_B \ll \eta_G$ holds true too. It follows that Eq. (20) may be valid even if the shear damping is high, say $\eta_G \approx 1.0$. From these, one can conclude that Eq. (21) may be accurate enough not only for the low loss materials, but also the high loss rubbers and other elastomers.

Eq. (21) shows that the Poisson's loss factor is directly proportional to the difference between the shear and bulk factor, which is the largest and smallest, respectively, amongst the modulus loss factors [8]. In addition, the magnitude of η_v depends on the dynamic Poisson's ratio too as defined by Eq. (21). Bearing in mind that the possible magnitudes of the dynamic Poisson's ratio are known from the theory, the investigation of the Poisson's loss factor as a function of ν_d deserves special attention, and it will be the subject of the next section.

2.3. Poisson's loss factor vs. dynamic Poisson's ratio

The magnitude of the Poisson's loss factor at $\nu_d = 0$ and 0.5, respectively, is easy to determine. It can be seen from both Eqs. (17) and (21) that $\eta_\nu = 0$ if $\nu_d = 0.5$. The physical reason of the vanishing Poisson's loss factor is clear in this case, namely the material is perfectly *incompressible* ($\bar{B} = \infty$) if $\nu_d = 0.5$, and then the lateral strain must vary in phase with the axial strain in each and every moment ($\delta_\nu = 0$). Similarly, it is also clear from the physical meaning of the Poisson's loss factor that $\eta_\nu = 0$ if $\nu_d = 0$ (no lateral motion, no phase lag). It should be noted that the latter statement is not plausible mathematically at first glance, because Eq. (5) defining η_ν yields an indefinite 0/0 form as $\nu_d \to 0$ due to the fact that then the loss part, ν_l, in the complex Poisson's ratio vanishes too. Nevertheless, it can be proved (e.g., by means of Eq. (9)) that this 0/0 form approaches zero as $\nu_d \to 0$. It is worth noting that, in both cases discussed above, the Poisson's loss factor vanishes regardless of the magnitude of the shear damping.

The magnitude of the Poisson's loss factor also can easily be determined if the dynamic Poisson's ratio is close to 0 and 0.5, respectively, by approximating Eq. (21). For the sake of convenience, the Poisson's loss factor related to the shear loss factor is considered, i.e.,

$$\frac{\eta_\nu}{\eta_G} \approx \left(1 - \frac{\eta_B}{\eta_G}\right)\frac{(1+\nu_d)(1-2\nu_d)}{3\nu_d}, \tag{22}$$

where $0 \leqslant \eta_B/\eta_G \leqslant 1$ [8,9].

The simplest case to determine η_ν/η_G is if ν_d is close to 0.5, i.e., the material is nearly incompressible like rubbers. Then, the bulk loss is much smaller than the shear loss as mentioned before, and, therefore, the approximation:

$$\left(1 - \frac{\eta_B}{\eta_G}\right)\frac{1+\nu_d}{3\nu_d} \approx 1 \tag{23}$$

holds true. Consequently, Eq. (22) can be written as

$$\frac{\eta_\nu}{\eta_G} \approx 1 - 2\nu_d. \tag{24}$$

It follows that the Poisson's loss factor related to the shear one, in case of near-incompressibility, essentially depends only on the dynamic Poisson's ratio.

If ν_d is close to zero, Eq. (22) can be approximated as

$$\frac{\eta_\nu}{\eta_G} \approx \left(1 - \frac{\eta_B}{\eta_G}\right)\frac{1}{3\nu_d}. \tag{25}$$

This equation formally seems to approach infinity as $\nu_d \to 0$, however, in this case η_ν must approach zero. Nevertheless, Eq. (25) is able to describe the vanishing of η_ν as $\nu_d \to 0$, because then $\eta_B \to \eta_G$. (The latter is clear from Eq. (14).) It can be concluded from the afore-mentioned that the prediction of Eq. (25) is adequate for $\nu_d \to 0$ if the member $(1-\eta_B/\eta_G)$ can be expressed as a function of ν_d, and this function obeys a power law defined as

$$1 - \frac{\eta_B}{\eta_G} = a\nu_d^n, \tag{26}$$

where $a > 0$ and $n > 1$ should stand. The replacement of Eq. (26) into Eq. (25) results in:

$$\frac{\eta_\nu}{\eta_G} \approx \frac{a}{3}\nu_d^{n-1}. \tag{27}$$

Eq. (27) predicts that the magnitude of η_B/η_G depends only on the dynamic Poisson's ratio if the latter is close to zero. The same conclusion has been drawn for the case when ν_d is close to 0.5. It is an intriguing possibility that the relation between η_B/η_G and ν_d may exist for other values of dynamic Poisson's ratio. Such a

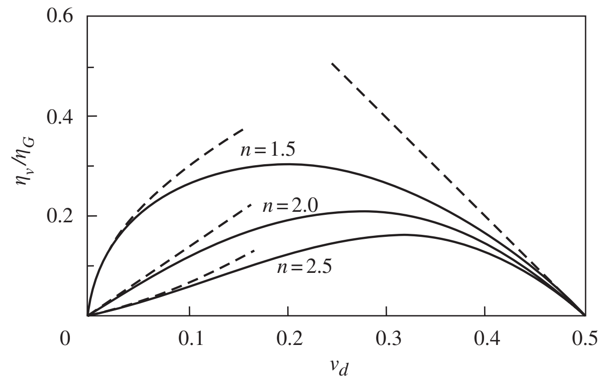

Fig. 1. The Poisson's loss factor related to the shear loss factor as a function of dynamic Poisson's ratio. —, predictions by Eq. (28) for $n = 1.5$, 2.0 and 2.5; - - -, predictions by Eqs. (24) and (27) for $\nu_d \to 0.5$ and $\nu_d \to 0$, respectively.

relation can simply be constructed by assuming that Eq. (26) holds true for all values of ν_d between 0 and 0.5. The investigation of this assumption, i.e., that the ratio of bulk to shear loss can be given as a function of ν_d, is the subject of a recent research, which definitely supports this idea, some results will be published soon. The replacement of Eq. (26) into Eq. (22) yields:

$$\frac{\eta_\nu}{\eta_G} \approx \frac{a}{3}\nu_d^{n-1}(1+\nu_d)(1-2\nu_d), \tag{28}$$

where $a = 2$, which follows from Eq. (26) seeing that $\eta_B \to 0$ as $\nu_d \to 0.5$.

At this stage, it is useful to summarize the assumptions used to develop Eq. (28). The assumptions are: (a) the solid material is homogeneous and isotropic, (b) the dynamic behaviour is linear, (c) the dynamic Poisson's ratio is positive, and $0 \leqslant \nu_d \leqslant 0.5$, (d) the shear damping is low, namely $\eta_B < 0.3$, (e) Eq. (26) holds true, i.e., the ratio of the bulk loss factor to the shear loss factor obeys a power law of the dynamic Poisson's ratio.

The variations of η_ν/η_G predicted by Eq. (28) are shown in Fig. 1 as a function of ν_d for $n = 1.5$, 2.0 and 2.5. Also are shown, for the sake of completeness, the approximations by Eqs. (24) and (27). It can be seen that η_ν/η_G varies along an arched curve, and Eq. (28) satisfies all requirements concerning η_ν, namely; $\eta_\nu = 0$ at both $\nu_d = 0$ and 0.5 as discussed before, and $\eta_\nu < \eta_G$ in agreement with previous theoretical studies [8,9]. Moreover, combining Eqs. (26) and (28) shows that $\eta_\nu < \eta_B$, which is also in agreement with the theoretical findings [8].

It should be emphasized, however, that it is not thought at all that Eq. (28) (and Eq. (26)), with certain coefficients a and n, would universally be valid for all solid materials over the whole range of dynamic Poisson's ratio. In contrast, the assumption is that Eq. (28) is able to relate η_ν/η_G to ν_d for real solids over restricted ranges of ν_d, and the coefficients a and n certainly vary from material to material. The adequacy of Eq. (28) will be investigated in the following paragraph by means of experimental data covering some ranges of ν_d.

3. Experimental data

3.1. Polyurethane rubber ($0.490 < \nu_d < 0.497$)

It is well known that the rubbers and other elastomers are nearly incompressible at low frequencies, in the rubbery range of dynamic behaviour, where $\nu_d \cong 0.5$. Notwithstanding, the rubbers loose this property with increasing frequency, and become compressible in the transition and glassy ranges, where the dynamic Poisson's ratio may be as low as 0.4 or 0.3 [3,5,8]. Consequently, the rubbers offer an excellent possibility to study the validity of both Eqs. (24) and (28). Unfortunately, only few experimental data on the Poisson's loss factor of rubbery materials are available from the technical literature [8,24], and even these data cannot be used for our purpose, because they are not complete or reliable enough. Bearing in mind that the complex

Poisson's ratio can be determined from the measurements of two complex moduli, it was decided to search for such experimental works, and to calculate η_v and ν_d from the available modulus data.

Detailed, reliable experimental data on the complex shear and bulk moduli of a commercial polyurethane rubber are in the paper by Mott et al. [25]. The complex shear modulus was measured as a function of frequency (10^{-4}–2 Hz) and temperature (−36 to 34 °C), mainly in the rubbery range and in the lower part of the rubber-to-glass transition range, and reduced frequency curves from 10^{-4} to 10^9 Hz were constructed at reference temperature 34 °C by means of the frequency–temperature equivalence principle. The complex bulk modulus was determined from the measurement of longitudinal sound speed and attenuation as a function of frequency (12.5–75 kHz) and temperature (3.9–32.6 °C). The combination of the longitudinal data with the shear data, and the application of the frequency-equivalence principle at reference temperature 34 °C, have resulted the reduced frequency curves for the complex bulk modulus from 10^4 to 3×10^7 Hz. The frequency variations of the dynamic shear and bulk moduli, and the respective loss factors, determined from the smoothed reduced frequency curves (Figs. 7 and 10 in Ref. [25]), are shown in Fig. 2.

In this work, the components of the complex Poisson's ratio were calculated from the experimental shear and bulk data below the loss factor peak, at frequencies 10^4, 10^5, 10^6 and 10^7 Hz. The Poisson's loss factor was calculated by Eq. (21), and the dynamic Poisson's ratio was determined by the formula:

$$\nu_d \approx \frac{3B_d - 2G_d}{6B_d + 2G_d}. \tag{29}$$

This equation can be derived from Eq. (9) under the condition that $\eta_G < 0.3$, which holds true at the frequencies of investigation. The calculated values of ν_d and η_v are given in Fig. 2. It can be seen that the dynamic Poisson's ratio decreases from 0.497 (10^4 Hz) down to 0.49 (10^7 Hz) in the frequency range of investigation, i.e., this polyurethane rubber is nearly incompressible at these frequencies. It can further be seen in Fig. 2 that the magnitude of the Poisson's loss factor is of the order of 10^{-3}, which is much smaller than that of the shear loss factor. The loss factors η_G and η_v, and the dynamic Poisson's ratio determined for this polyurethane rubber were used to verify Eq. (24). The calculated values of η_v/η_G are given in Fig. 3 as functions

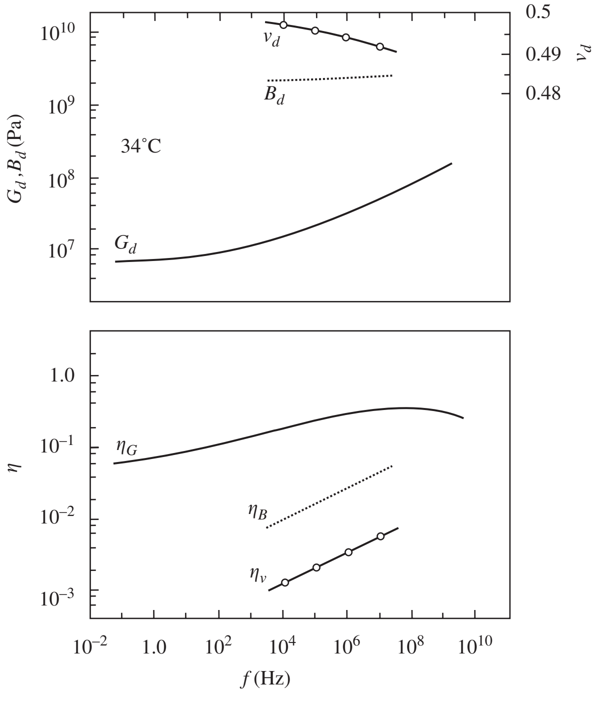

Fig. 2. Dynamic properties of a polyurethane rubber plotted against the frequency. The shear (—) and bulk (· · · ·) properties are from the experimental work by Mott et al. [25]. The dynamic Poisson's ratio and the respective loss factor (○) are calculated from the shear and bulk data at the frequencies as shown in the figure.

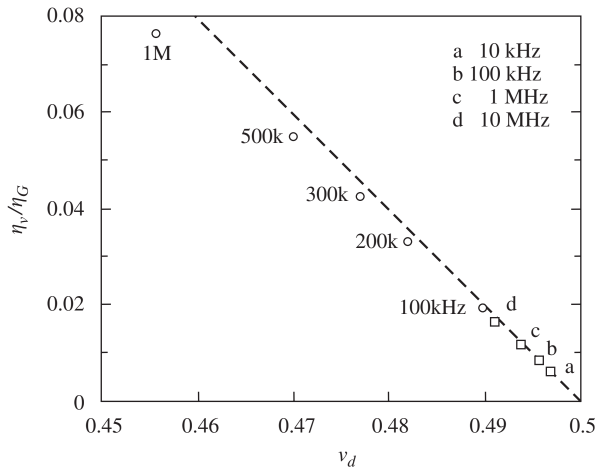

Fig. 3. The Poisson's loss factor related to the shear loss factor as a function of dynamic Poisson's ratio in case of near-incompressibility. - - -, prediction by Eq. (24); □, polyurethane rubber (34 °C); ○, styrene-butadiene rubber (20 °C). The experimental magnitudes of η_v/η_G are calculated from the data on η_v and η_G seen in Figs. 2 and 4, respectively.

of ν_d. In addition, the prediction of Eq. (24) is shown in Fig. 3. The agreement between the experimental data and the theoretical prediction is rather good.

3.2. Styrene-butadiene rubber ($0.416 < \nu_d < 0.49$)

The very careful and accurate measurements of the complex bulk modulus and the complex longitudinal (wave) modulus, $\bar{L}(j\omega)$, of a styrene-butadiene rubber performed by Wada et al. [26] offer a further possibility to investigate the validity not only of Eq. (24) but also Eq. (28). The authors of work [26] measured the complex bulk and longitudinal moduli at some frequencies in the ultrasonic range (0.33–5 MHz), as functions of temperature (0–55 °C), and calculated the complex shear modulus from these data. Reduced frequency curves were constructed for the dynamic and loss moduli at reference temperature 20 °C, over the frequency range from 10^4 to 10^9 Hz, which covers the main transition zone of this rubber. These data (Fig. 10 in Ref. [26]) were used to plot the dynamic bulk, longitudinal and shear moduli in Fig. 4. Also are given in Fig. 4 the respective loss factors, which were determined by the present author from the dynamic and loss moduli. Unfortunately, the shear properties below 10^5 Hz (broken line in Fig. 4) could not reliably be determined from the available experimental data.

The dynamic Poisson's ratio and loss factor were calculated from the bulk and longitudinal properties. This method was used earlier to determine the complex Poisson's ratio for this rubber, the equations of the calculations are [3]:

$$\nu_d \approx \frac{3B_d - L_d}{3B_d + L_d}, \tag{30}$$

$$\eta_\nu \approx (\eta_L - \eta_B)\frac{(1+\nu_d)(1-\nu_d)}{2\nu_d}, \tag{31}$$

where L_d is the dynamic longitudinal modulus and η_L is the respective loss factor. Eqs. (30) and (31) can be used if $\eta_L < 0.3$ and $\eta_B < 0.3$; both conditions are satisfied in this case. The calculated values of ν_d and η_ν are given in Fig. 4. It can be seen that $\nu_d \approx 0.5$ at low frequency, while ν_d approaches 0.4 at high frequency. It can further be seen that the Poisson's loss factor is much smaller than the shear one over the whole frequency range of investigation, and the larger the difference between them, the closer is ν_d to 0.5. The magnitudes of η_ν/η_G were calculated at some frequencies between 10^5 ($\nu_d = 0.49$) and 10^8 Hz ($\nu_d = 0.416$), the results are given in Figs. 3 and 5. It can be seen in Fig. 3 that the experimental data are in good agreement with the

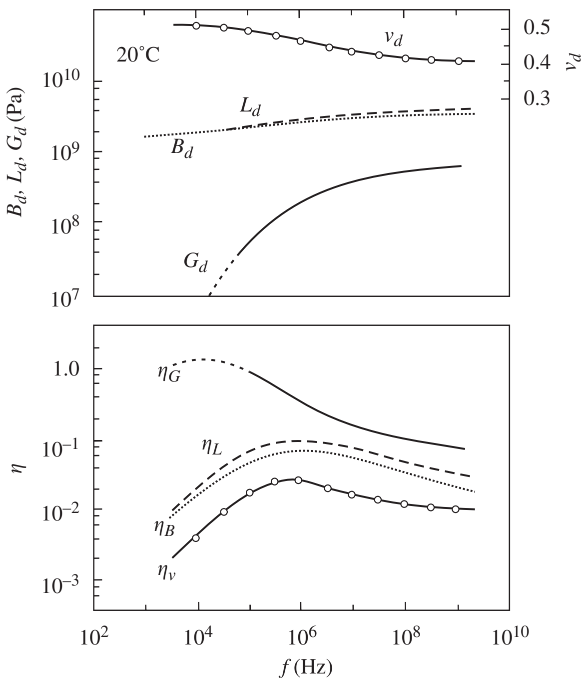

Fig. 4. Dynamic properties of a styrene-butadiene rubber plotted against the frequency. The shear (—), bulk (· · · ·) and longitudinal (- - -) properties are from the experimental work by Wada et al. [26]. The dynamic Poisson's ratio and the respective loss factor (○) are calculated from the bulk and longitudinal data at the frequencies as shown in the figure.

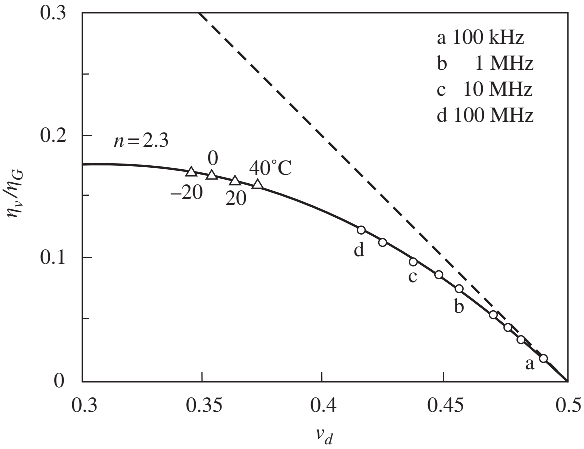

Fig. 5. The Poisson's loss factor related to the shear loss factor as a function of the dynamic Poisson's ratio. - - -, prediction by Eq. (24) for the case of near-incompressibility; —, prediction by Eq. (28) with $n = 2.3$. ○, styrene-butadiene rubber (20 °C). The experimental magnitudes of η_v/η_G are calculated from the data on η_v and η_G seen in Fig. 4. Δ, PMMA. The data on η_v/η_G and ν_d are from the measurements made by Yee and Takemori at 1 Hz (Figs. 4 and 7 in Ref. [10]).

prediction of Eq. (24) if $\nu_d > 0.48$, but the data deviate from that by decreasing ν_d. The increasing deviation is the consequence of the decreasing difference between the bulk and shear loss factors (Fig. 4). Nevertheless, Fig. 5 clearly demonstrates that, with decreasing ν_d, the experimental data follow the prediction of Eq. (28), and excellent fitting has been found with $n = 2.3$.

3.3. Poly(methyl methacrylate) ($0.345 < \nu_d < 0.372$)

In contrast to rubbers, the hard plastics were in focus in several experimental works to determine the Poisson's loss factor [1,8–11]. Of these works, the measurements made by Yee and Takemori [10] on poly(methyl methacrylate) (PMMA) are of special importance and considered here, because of the exceptionally high experimental accuracy and reliability of the results. The high accuracy is partly due to the direct method used by the authors to measure the components of the complex Poisson's ratio. In addition to ν_d and η_ν, the complex Young's modulus was measured on one and the same specimen, and the complex shear and bulk moduli were calculated from the measured data. The measurements were made at 1 Hz as a function of temperature in the range from −40 to 100 °C, where the so-called secondary or β transition of PMMA occurs. The Poisson's loss factor passes through a maximum, while the dynamic Poisson's ratio decreases with increasing temperature, as expected. The maximum in η_ν is shallow, and occurs at around 20 °C, where $\eta_\nu = 0.013$, $\nu_d = 0.363$ and $\eta_G = 0.079$; these values are typical for hard plastics at room temperature [1,8–11].

The experimental data on ν_d, η_ν and η_G at temperatures −20, 0, 20 and 40 °C (Figs. 4 and 7 in Ref. [10]) were used to verify Eq. (28). The calculated values of η_ν/η_G are given in Fig. 5, as functions of ν_d ranging from 0.345 to 0.372. It can be seen that the experimental data, like the styrene-butadiene rubber, fit very well to the prediction of Eq. (28) if $n = 2.3$.

Although the experimental data seen in Fig. 5 convincingly support the theoretical predictions, more data are required, of course, to verify Eq. (28). Unfortunately, such data, especially for the lower values of ν_d, are not available or some of the published data cannot be used for this purpose, because they are not complete (η_G or ν_d is not known) or are not reliable enough due to experimental difficulties. The measurement of the Poisson's loss factor for materials of different kind, and the investigation of Eq. (28) by experimental data offer an exciting research topic.

3.4. Comparison of the experimental data

Up to now, the magnitudes of η_ν/η_G were in focus to verify the equations developed in the paper. In Fig. 6 the magnitudes of η_ν are plotted against ν_d for the three materials discussed above. It can be seen that the experimental data form an inverted "U" plot (in case of the polyurethane rubber, the data are not enough to see this plot.) This inverted "U" plot is well known from the measurements of complex modulus of viscoelastic damping materials, when the modulus loss factor is plotted against the dynamic modulus [27]. Notwithstanding, there is a distinct difference between the inverted "U" plots concerning a complex modulus

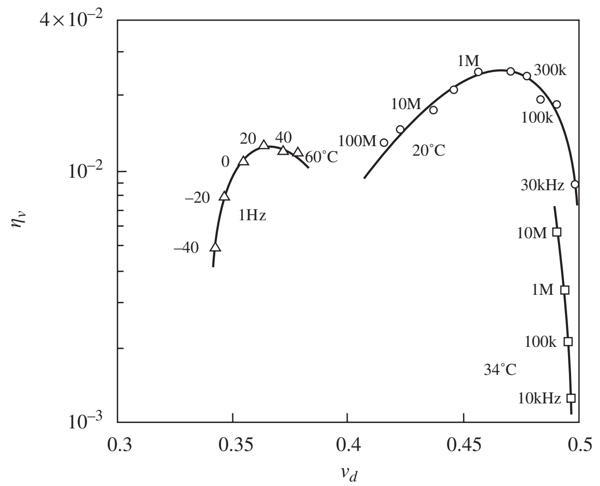

Fig. 6. The magnitudes of Poisson's loss factor plotted against the dynamic Poisson's ratio. □, polyurethane rubber (data from Fig. 2); ○, styrene-butadiene rubber (data from Fig. 4); Δ, PMMA (data from Figs. 4 and 7 in Ref. [10]).

and the complex Poisson's ratio, respectively. In case of a complex modulus, e.g., the shear one, the height of this plot, i.e., the highest magnitude of the modulus loss factor, is in direct relation to the damping ability of the material in question. In contrast, it can be seen in Fig. 6 that the highest magnitude of the Poisson's loss factor of the styrene-butadiene rubber ($\eta_v = 0.026$) and the PMMA ($\eta_v = 0.013$) do not differ significantly from each other, while their shear damping differs by more than one order of magnitude (the maximum in η_G of the styrene-butadiene rubber exceeds 1.0 (Fig. 4), and that of PMMA is 0.079 [10]). In addition, Fig. 6 clearly demonstrates that η_v of the high loss polyurethane rubber, in the range of near-incompressibility ($v_d > 0.49$), may be much smaller than η_v of the low loss PMMA. These phenomena are the consequence of the fact that the magnitude of η_v is dependent on the dynamic Poisson's ratio beside the material damping as formulated by Eq. (28).

4. Discussion

The essential aim of this work was to determine the magnitude of the Poisson's loss factor, and to clear up its relation to the material damping for homogeneous, isotropic, linear solid viscoelastic materials with positive Poisson's ratio. As a result of theoretical study, it has been found that the Poisson's loss factor is approximately proportional to the difference between the shear and bulk loss factors, moreover it is a function of the dynamic Poisson's ratio. In addition, it has been shown that the Poisson's loss factor can be approximately determined from knowledge only of the shear loss factor and the dynamic Poisson's ratio by means of the relationships developed in the paper. These relationships have been verified by experimental data on two rubbers and a hard plastic (PMMA) in the range of dynamic Poisson's ratio from 0.345 to 0.497.

Of the relationships developed in the paper, Eq. (28) is of special interest and usefulness. The usefulness of Eq. (28) is in that it enables one to estimate the magnitude of the Poisson's loss factor for solid materials, since the shear loss factor and the dynamic Poisson's ratio are more or less known for materials of different kind. In general, it can be proved through Eq. (28) that the Poisson's loss factor is smaller than the shear loss factor usually by one order of magnitude at least (Fig. 1). Clearly, among the solid materials, the high loss rubbers and other elastomers may have the highest Poisson's loss factor. It is known that the shear loss factor of these rubbery materials may be as high as 1.0 [1,5,6], and their dynamic Poisson's ratio is usually larger than 0.46…0.47 at and around the shear loss peak as can be seen in Figs. 2 and 4. From these one can conclude by means of Eq. (28) that magnitude of the Poisson's loss factor normally does not exceed 0.1 even if the shear damping is high. Notwithstanding, the Poisson's loss factor of a high loss rubbery material may be much smaller than 0.1 if v_d is close to 0.5 or zero as mentioned before. The highest magnitude of the Poisson's loss factor of hard plastics, by this estimation, is of the order of 10^{-3}–10^{-2}, seeing that the respective shear loss peak is of the order of 10^{-2}–10^{-1}, and the dynamic Poisson's ratio lays between approximately 0.3 and 0.4 [1,8–11]. Furthermore, it is clear from Eq. (28) that the Poisson's loss factor is inevitably low and can be extremely low for the low loss materials like metals, ceramics, glasses, etc. It is interesting to conclude from the above discussion that the magnitude of the Poisson's loss factor may be about the same for a high loss and a low loss material as can be seen in Fig. 6.

Although the Poisson's loss factor is always low, the highest experienced magnitude is of the order of 10^{-2} [1,8–12], this tiny quantity plays important role in the material behaviour. In spite of it, the Poisson's loss factor, referring to its smallness, is frequently neglected, however, it leads to contradiction in characterizing the dynamic behaviour of materials, and may cause erroneous results in the acoustical calculus. On one hand, the negligence of η_v implies that the shear loss factor is identical to the bulk and any other modulus loss factors (see, e.g., Eqs. (14), (21) and (31)), which is absurd and contradicts both solid theoretical statements [8,9] and experimental observations [1,8–10]. On the other hand, the assumption on $\eta_v = 0$ implies that the dynamic Poisson's ratio is independent of the frequency, whilst the frequency dependence is inevitable [2–4] and is in direct relation to the magnitude of η_v as can be seen from Eq. (8). No doubt that in some cases, especially with the low loss structural materials (metals, ceramics, etc.), the frequency dependence of v_d (and the magnitude of η_v) can be neglected, but in other cases, mainly with the rubbery materials widely used for vibration and sound control, the assumption on the frequency independent Poisson's ratio may cause serious error in the acoustical calculus. Namely, there are a number of equations, which contain the member $1/(1-2v_d)$, and, therefore, these equations are highly sensitive to the magnitude of, i.e., any small frequency variation in v_d, if it is close to 0.5.

The importance of this question is demonstrated by a numerical example. Consider a frequency range covering three decades from ω_1 to ω_2, where $v_d(\omega_1) = 0.49$ and $\eta_v = 0.005$; the latter is assumed to be constant over this range for the sake of simplicity. In this case, the frequency variation in $v_d(\omega)$ can be predicted from η_v by means of Eq. (8), from which one can derive that:

$$v_d(\omega_2) \approx v_d(\omega_1)\left(\frac{\omega_1}{\omega_2}\right)^{2\eta_v/\pi}. \tag{32}$$

Using the above data, Eq. (32) yields: $v_d(\omega_2) = 0.4793$, i.e., the decrease in $v_d(\omega)$ from ω_1 to ω_2 is about 2%, which seems to be negligible from practical point of view. Nevertheless, if this small variation in $v_d(\omega)$ is neglected, i.e., $\eta_v = 0$ is assumed, then the error in the calculation by an equation containing the member $1/(1-2v_d)$, may exceed 100% at ω_2.

5. Conclusions

The magnitude of the Poisson's loss factor has been investigated in this paper for homogeneous, isotropic, linear solid viscoelastic materials with positive dynamic Poisson's ratio. As a result of theoretical investigation and the experimental data presented in the paper, the following conclusions can be drawn:

(a) The magnitude of the Poisson's loss factor is approximately proportional to the difference between the shear and bulk loss factors, and is a rational fractional function of the dynamic Poisson's ratio.
(b) The Poisson's loss factor can be approximately determined from knowledge only of the shear loss factor and the dynamic Poisson's ratio by means of the relationships developed in the paper.
(c) The Poisson's loss factor is smaller than the shear loss factor usually by one order of magnitude at least.
(d) The magnitude of the Poisson's loss factor is low and normally does not exceed 0.1 even in the case of rubbers and other elastomers characterized by high shear damping.
(e) The Poisson's loss factor of a high loss and a low loss material may be about the same.

Acknowledgements

This work is a part of a research project supported by the Hungarian Scientific Research Fund (OTKA) under contract T 37313. The financial support is gratefully acknowledged.

References

[1] B.E. Read, G.D. Dean, *The Determination of Dynamic Properties of Polymers and Composites*, Adam Hilger, Bristol, 1978.
[2] N.W. Tschoegl, W.G. Knauss, I. Emri, Poisson's ratio in linear viscoelasticity—a critical review, *Mechanics of Time-Dependent Materials* 6 (2002) 3–51.
[3] T. Pritz, Measurement methods of complex Poisson's ratio of viscoelastic materials, *Applied Acoustics* 60 (2000) 279–292.
[4] T. Pritz, Frequency dependences of complex moduli and complex Poisson's ratio of real solid materials, *Journal of Sound and Vibration* 214 (1998) 83–104.
[5] D.I.G. Jones, *Handbook of Viscoelastic Vibration Damping*, Wiley, New York, 2001.
[6] J.D. Ferry, *Viscoelastic Properties of Polymers*, third ed., Wiley, New York, 1980.
[7] L.D. Landau, E.M. Lifshitz, *Theory of Elasticity*, Pergamon, Oxford, 1970.
[8] P.S. Theocaris, Interrelation between dynamic moduli and compliances in polymers, *Kolloid-Zeitschrift und Zeitschrift fürPolymere* 235 (1969) 1182–1188.
[9] H.A. Waterman, Relations between loss angles in isotropic linear viscoelastic materials, *Rheologica Acta* 16 (1977) 31–42.
[10] A.F. Yee, M.T. Takemori, Dynamic bulk and shear relaxation in glassy polymers. I. Experimental techniques and results on PMMA, *Journal of Polymer Science: Polymer Physics Edition* 20 (1982) 205–224.
[11] M. Giovagnoni, On the direct measurement of the dynamic Poisson's ratio, *Mechanics of Materials* 17 (1994) 33–46.
[12] L. Boeckx, P. Leclaire, P. Khurana, C. Glorieux, W. Lauriks, Investigation of the phase velocities of guided acoustic waves in soft porous layers, *Journal of the Acoustical Society of America* 117 (2005) 545–554.
[13] D.J. Mead, R.J. Joannides, Measurement of the dynamic moduli and Poisson's ratio of a transversely isotropic fibre-reinforced plastic, *Composites* 22 (1991) 15–29.

[14] P.S. Dubbelday, Poisson's ratio of foamed aluminium determined by laser Doppler vibrometry, *Journal of the Acoustical Society of America* 91 (1992) 1737–1744.
[15] S. Sahraoui, E. Mariez, M. Etchessahar, Mechanical testing of polymeric foams at low frequency, *Polymer Testing* 20 (2001) 93–96.
[16] S. Mousavi, D.F. Nicolas, B. Lundberg, Identification of complex moduli and Poisson's ratio from measured strains on an impacted bar, *Journal of Sound and Vibration* 277 (2004) 971–986.
[17] R.L. Willis, L. Wu, Y. Berthelot, Determination of the complex Young and shear dynamic moduli of viscoelastic materials, *Journal of the Acoustical Society of America* 109 (2001) 611–621.
[18] A. Di Meglio, L.S. Wang, A variational method for identification of viscoelastic parameters from experimental data, *Journal of the Acoustical Society of America* 108 (2000) 2746–2753.
[19] S.G. Hutton, G. Chen, Determination of the complex moduli of viscoelastic isolator mounts by finite element modelling and vibration testing, *International Journal of Acoustics and Vibration* 3 (1998) 103–108.
[20] V.V. Mokeyev, A generalized complex eigenvector method for dynamic analysis of heterogeneous viscoelastic structures, *International Journal for Numerical Methods in Engineering* 50 (2001) 2271–2282.
[21] N. Dauchez, M. Etchessahar, S. Sahraoui, On measurement of mechanical properties of sound absorbing materials, *Proceedings of Second Biot Conference on Poromechanics*, Grenoble, 2002, pp. 627–631.
[22] T. Pritz, Verification of local Kramers–Kronig relations for complex modulus by means of fractional derivative model, *Journal of Sound and Vibration* 228 (1999) 1145–1165.
[23] R.S. Lakes, Foam structures with a negative Poisson's ratio, *Science* 235 (1987) 1038–1040.
[24] K.C. Thomson, On the complex Poisson's ratio of a urethane rubber compound, *Journal of Applied Polymer Science* 10 (1966) 1133–1136.
[25] P.H. Mott, C.M. Roland, R.D. Corsaro, Acoustic and dynamic mechanical properties of a polyurethane rubber, *Journal of the Acoustical Society of America* 111 (2002) 1782–1790.
[26] Y. Wada, R. Ito, H. Ochiai, Comparison between mechanical relaxation associated with volume and shear deformations in styrene-butadiene rubber, *Journal of the Physical Society of Japan* 17 (1962) 213–218.
[27] ISO 10112, Damping materials—Graphical presentation of the complex modulus, 1991.